Math 10B with Professor Stankova
Quiz 9; Tuesday, 3/20/2018
Section #211; Time: 11 AM
GSI name: Roy Zhao **Name:** ______________________

Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. **TRUE** False A solution to an IVP may not always exist nor be unique.

> **Solution:** If the function $f(y, t)$ is not continuous, then the IVP may not exist and even if it is continuous, then it may not be unique.

2. True **FALSE** In order to verify that $a_n = f(n)$ is a solution to a recurrence equation, we need to solve the recurrence equation and see if it matches with $f(n)$.

> **Solution:** We just need to plug in $f(n)$ for a_n and check that the equation is satisfied.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (4 points) Find the solution to $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 0, a_1 = -3$.

> **Solution:** The characteristic equation is $\lambda^n = \lambda^{n-1} + 2\lambda^{n-2}$ and simplifying gives $\lambda^2 - \lambda - 2 = (\lambda - 2)(\lambda + 1) = 0$. Thus, the roots are $\lambda = -1, 2$ and hence the general solution is $a_n = c_1(-1)^n + c_2 2^n$. Now we plug in the initial conditions gives $a_0 = c_1 + c_2 = 0$ and $a_1 = -c_1 + 2c_2 = -3$ and hence $c_1 = 1, c_2 = -1$. So the solution is $a_n = (-1)^n - 2^n$.

(b) (2 points) Verify that $3n + 1$ is a solution to $a_n = 2a_{n-1} - a_{n-2}$.

> **Solution:** We just need to plug in the answer and make sure the equality is satisfied. Doing so gives us $3n + 1 = 2(3(n - 1) + 1) - (3(n - 2) + 1) = 6n - 6 + 2 - 3n + 6 - 1 = 3n + 1$ as required.

(c) (4 points) Find the general solution to $y' + y = e^{-t}$.

> **Solution:** The integrating factor that we need to multiply by is $I(t) = e^{\int 1dt} = e^t$ and doing so gives us $e^t y' + e^t y = (e^t y)' = 1$. Integrating gives $e^t y = t + C$ and hence $y = te^{-t} + Ce^{-t}$ as the general solution.